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CALCULATION OF RADIATIVE-CONDUCTIVE HEAT TRANSFER IN A SEMITRANSPARENT PLATE BY THE MONTE CARLO METHOD

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The possibility of applying the Monte Carlo method together with the method of finite differences for calculating radiative–conductive heat transfer is analyzed.

At the present time, a great deal of attention is devoted to the study of radiative–conductive heat transfer. The energy equation in this case is a nonlinear integrodifferential equation. Its solution had not been found in general. A series of papers [1-3] is devoted to the numerical solution of this equation in different particular cases. In the general case, in taking into account the nongray nature of the medium and the temperature dependences of the absorption coefficient and thermophysical parameters, it is difficult to obtain a numerical solution using traditional methods.

In such complex cases, it is expedient to use the Monte Carlo method. The essence of this method consists in constructing for a given problem a random process whose parameters are the quantities sought. They are determined by observing the random process and calculating its statistical characteristics, approximately equal to the parameters sought.

Radiative heat transfer in semitransparent media was studied in [4, 5] using the Monte Carlo method. In this paper, we examine the more general form of heat transfer: radiative–conductive. In so doing, in order to carry out the calculations, we attempt to combine the Monte Carlo method with a finite-difference numerical method. The analysis is based on the scheme for the Monte Carlo method used in [4].

We examined nonstationary radiative–conductive heat transfer in the absence of scattering. Such heat transfer occurs, for example, in processes of glass formation. The problem was to determine the temperature fields for two types of glasses: white and green. We calculated the cooling of an infinite plate taking into account the wavelength dependence of the coefficient of absorption and the temperature dependence of the thermophysical parameters. These dependences were obtained by approximating the data in [6].

The system of equations, describing heat transfer, in this case has the form

$$\frac{\partial I_\lambda}{\partial x} = \beta_\lambda \left[\frac{e_{b\lambda}(x)}{\pi} - I_\lambda\right], \tag{1}$$

$$\frac{\partial}{\partial t}(\rho c_p T) = -4\int_0^\infty \beta_\lambda e_{b\lambda}(x)\,d\lambda + 2\pi\int_{-1}^{+1}\mu d\mu\int_0^\infty \beta_\lambda I_\lambda d\lambda + \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right). \tag{2}$$

In view of the absolute contact between the glass layer and nontransparent surfaces, boundary conditions of the first kind T_s are given. The temperature T_i, identical for the entire glass specimen, is given as an initial condition.

In order to solve the system (1-2), the plate was separated into zones (Fig. 1), while the cooling process was separated into time intervals. In order to determine the intensity

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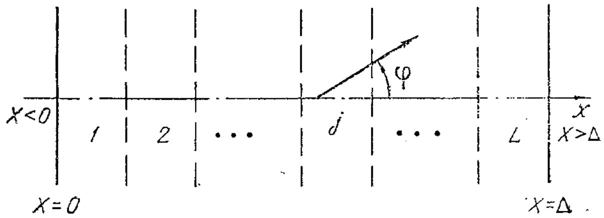

Fig. 1. Scheme for separating the plate into zones: 1, 2, ..., j, ..., L, are the zone numbers; Δ, plate thickness; φ, angle between the direction of motion of the particle and the x axis, $\mu = \cos\varphi$.

of radiation, we simulated the process of photon emission at a given temperature distribution, which was assumed to be constant at each time interval everywhere in the zone examined. The radiation intensity obtained in each zone was used to calculate the temperature at the next time interval using Eq. (2). The latter was replaced by a system of finite-difference equations, whose solution yields the temperature of each zone. Then, the entire procedure was repeated in the next time interval.

Particle generation was examined for two types of sources:

1) surface

$$S_i^s = \varepsilon \int\limits_{\lambda_i}^{\lambda_{i+1}} e_{b\lambda} d\lambda = \varepsilon n^2 \sigma T_s^4 F_{nT\lambda_i - nT\lambda_{i+1}}, \tag{3}$$

2) volume

$$S_{ij}^k = 4\Delta x \int\limits_{\lambda_i}^{\lambda_{i+1}} \beta_i e_{b\lambda} d\lambda = 4\Delta x \beta_i n^2 \sigma (T_j^k)^4 F_{nT\lambda_i - nT\lambda_{i+1}}, \tag{4}$$

$$F_{nT\lambda_i - nT\lambda_{i+1}} = \frac{1}{\sigma} \int\limits_{nT\lambda_i}^{nT\lambda_{i+1}} \frac{e_{b\lambda}}{n^3 T^5} d(n\lambda T), \tag{5}$$

where $F_{nT\lambda_i - nT\lambda_{i+1}}$ is the fraction of the blackbody radiation emitted in the spectral band $\lambda_i - \lambda_{i+1}$. In order to determine it, the following polynomial dependences are usually used [7]:

$$F_{0-nT\lambda} = \frac{15}{\pi^4} \sum_m \frac{\exp(-m\vartheta)}{m^4} \{[(m\vartheta + 3) m\vartheta + 6] m\vartheta + 6\} \quad \text{for} \quad \vartheta \geqslant 2,$$

$$F_{0-nT\lambda} = 1 - \frac{15}{\pi^4} \vartheta^3 \left(\frac{1}{3} - \frac{\vartheta}{8} + \frac{\vartheta^2}{60} - \frac{\vartheta^4}{5040} + \frac{\vartheta^6}{272160} - \frac{\vartheta^8}{13305600}\right) \quad \text{for} \quad \vartheta < 2, \tag{6}$$

where $\vartheta = c_2/(nT\lambda)$ and $c_2 = 1.4387 \cdot 10^{-2}$ mK°.

The number of particles emitted by the j-th zone at the k-th time interval was determined using a two-step procedure. First, the number of particles emitted in the i-th wavelength interval was found:

$$N_i^k = E\left[\frac{N\left(S_i^s + \sum_j S_{ij}^k\right)}{\sum_i S_i^s + \sum_i \sum_j S_{ij}^k}\right], \tag{7}$$

where E[...] indicates the integer part of the enclosed number. Then, the number of surface and volume particles was established:

$$N_{in}^{k} = E\left[\frac{N_i^k S_i^s}{S_i^s + \sum_j S_{ij}^k}\right], \tag{8}$$

$$N_{ij}^{k} = E\left[\frac{N_i^k S_{ij}^k}{S_i^s + \sum_j S_{ij}^k}\right]. \tag{9}$$

Each such particle was identified with a beam of photons, whose energy was determined from the expressions:

$$E_{is}^{k} = S_i^s \Delta t / N_{is}^k, \tag{10}$$

$$E_{ij}^{k} = S_{ij}^k \Delta t / N_{ij}^k. \tag{11}$$

Since $C\Delta t \gg \Delta$, at the end of each time interval, all particles generated are either absorbed or leave the plate.

Besides the particle energy, it is necessary to establish the position of their sources and the flight direction. These quantities are given by the following relations [4]: for "surface" particles

$$x = 0, \quad \mu = \sqrt{R} \text{ and } x = \Delta, \quad \mu = -\sqrt{R}, \tag{12}$$

for "volume" particles

$$x = (j - 1 + R)\,\Delta x, \quad \mu = 2R - 1, \tag{13}$$

where R is a random quantity, uniformly distributed in the interval (0, 1).

The distance traversed by particles up to the time of absorption is

$$d = \frac{|\ln R|}{\beta_i}. \tag{14}$$

Then, the number of zones in which absorption occurred was determined:

$$j' = E\,[x + d\mu]/\Delta x + 1. \tag{15}$$

If $j' > L$ or $j' < 0$, then the following condition was checked:

$$q = R - \varepsilon. \tag{16}$$

If q turned out to be less than 0, then it was assumed that the particle left the specimen, while in the opposite case, the particle was assumed to have been reflected from the wall, and its coordinates and direction were determined as for the surface particles and the calculation was repeated.

After examining the history of all particles, the energy balance in each zone was determined:

$$\Delta E_j^k = \sum_i N_{ij}^k - \sum_i \sum_{m=1}^{h} E_{im}^k, \tag{17}$$

where h is the number of particles absorbed in the j-th zone at the k-th time interval.

The temperature of the j-th zone was determined from the following relation, representing Eq. (2) written in finite-difference form:

$$T_j^{k+1} = T_j^k - \frac{1}{\rho_j^k c_{pj}^k \Delta x}\,\Delta E_j^k + \frac{K_j^k}{\rho_j^k c_{pj}^k \Delta x^2}\,[T_{j+1}^k - 2T_j^k + T_{j-1}^k]. \tag{18}$$

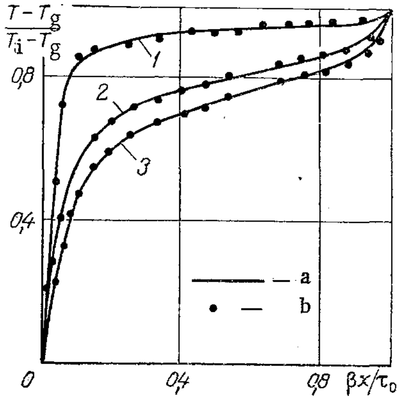

Fig. 2. Curves showing the temperature distribution in a gray absorbing gas according to the results obtained in [2] (a) and using the Monte Carlo method (b): 1) Fo = 0.001; 2) 0.005; 3) 0.01.

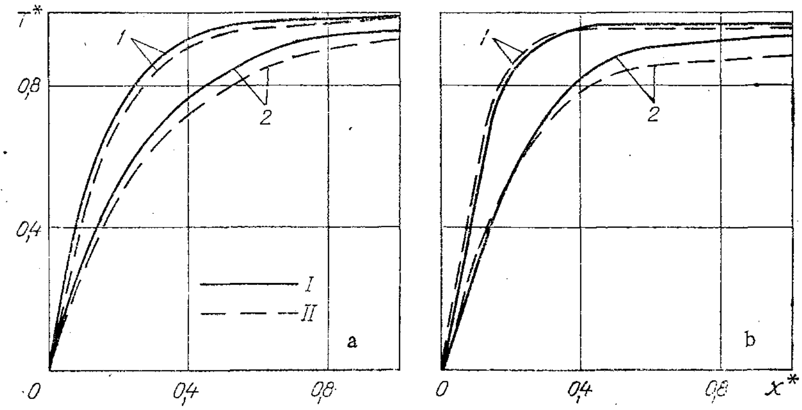

Fig. 3. Curves of the distribution of reduced temperatures in a plate made of green and white (b) glass using the results obtained by the Monte Carlo method (I) and the finite-difference method (gray approximation) (II). a: 1) t = 6 sec; 2) 26; b: t = 10 sec; 2) 30.

After determining the temperature distribution in the plate at the (k + 1)-st time interval, all calcultions are again repeated for the next time step.

The temperature fields for the specimens indicated above were calculated on an ES-1022 computer. The pseudorandom numbers were generated using the algorithm in [8], which has a sequence length for random numbers equal to 2^{33}, modified for use on the given computer.

In order to check the method and to determine the errors, we calculated the nonstationary radiative-conductive heat transfer in a gray absorbing gas for the case examined in [2] with $\tau_0 = 1$, P 0.005, $\Theta = 0.5$, and $\varepsilon = 1$. The results of the calculation are presented in Fig. 2 for different values of the Fourier number Fo.

One feature of the Monte Carlo method, which is common to any statistical method, is the fact that the error can be estimated from the probability. Examining Eq. (18), it can be shown that when the number of particles N, generated in each time interval, increases, the temperature T_j^k approaches in probability to the true value of the temperature in the j-th zone.

Assuming that the variances determining the temperatures in neighboring zones at a given time interval approximately equal the correlation moments of the temperatures of these zones, we obtain

$$D[T_j^{k+1}] = D[T_j^k] + \left(\frac{1}{\rho_j^k c_{pj}^k \Delta x}\right)^2 D[\Delta E_j^k], \tag{19}$$

where D[z] is the variance of the random quantity z. Since for k = 0, $D[T_j^\circ] = 0$ (the initial temperature is given), it is possible to obtain

$$D[T_j^{k+1}] = \sum_{m=1}^{k} \left(\frac{1}{\rho_j^m c_{pj}^m \Delta x}\right) D[\Delta E_j^m]. \tag{20}$$

The quantity $D[\Delta E_j^m]$ was calculated during the calculations as the sample variance of the term $\sum_i \sum_m E_{im}^k$ from Eq. (17). In addition, at each time interval, the total energy of the particles traversing a given zone was determined. Since Eq. (18) contains the term ΔE_j^k, consisting of a sum of a large number of random quantities, we find that

$$T_j^k - l_\xi \sqrt{\frac{D[T_j^k]}{M_j^k - 1}} < T_{\text{true}} < T_j^k + l_\xi \sqrt{\frac{D[T_j^k]}{M_j^k - 1}} \tag{21}$$

with probability ξ, where T_{true} is the true value of the temperature of the j-th zone at the k-th time interval; l_ξ is the root of the equation

$$\int_{-l}^{+l} W_{M-1}(y)\, dy = \xi, \tag{22}$$

where $W_{M-1}(x)$ is the Student probability distribution density with M − 1 degrees of freedom.

Choosing the confidence coefficient ξ = 0.999, for values of M encountered in the calculation, we find from [9] $l_\xi = 3.291$. The maximum value of the confidence interval for N = 10,000 particles, used in the calculation, did not exceed $\pm 0.01T_i$.

Using the procedure developed, we calculated the temperature fields for symmetrically cooled plates made of white and green glass. The initial temperature of the glass was assumed to be equal to 1400°K and the thickness of the plates was Δ = 45 mm. The temperature of the bounding gray surfaces, assumed to be diffuse, was 750°K and the emissivity was ε = 0.5.

The plates were separated into 45 zones. The number of zones was chosen from the condition that the coordinate discretization error did not exceed $0.01T_i$. The size of the interval was 0.4 sec. The number of particles, generated at each step in time, was chosen as equal to N = 10,000, which ensured for a given coordinate and time step a confidence interval and temperature not exceeding $0.01T_i$. In this case, 20 sec of machine time were needed to calculate a single time step.

The results of the calculation of temperature fields for different times are shown in Fig. 3. The reduced temperature $T^* = (T - T_s)/(T_i - T_s)$ is shown along the ordinate axis and the relative thickness of the plates $x^* = 2x/\Delta$ is shown along the abscissa axis.

Analysis of Fig. 3 shows that the values of the temperature obtained using the Monte Carlo method near the axis of the plates are somewhat higher than that obtained using the finite-difference numerical method. This can be explained by the fact that the wavelength dependence of the coefficient of absorption was taken into account in the first case.

The results obtained show that the Monte Carlo method can be used to calculate radiative–conductive heat transfer with high accuracy.

NOTATION

X, coordinate axis; λ, wavelength; μ, cosine of the angle between the direction of motion of the particles and the x axis; $e_{b\lambda}$, spectral emissivity of a black body; j, zone number; L, number of zones into which the specimen is divided; i, j, and k, indices indicating the wavelength interval from λ to λ_{i+1}, the j-th zone, and the k-th time interval to which a parameter belongs, respectively; ρ, density; c_p, heat capacity; K, coefficient of thermal conductivity; T, absolute temperature; ε, emissivity of the surfaces bounding the plates; S^s and S, flux density of emission by the bounding surfaces and the plates; σ, Stefan–Boltzmann constant; β, average absorption coefficient; N, number of particles emitted; M, number of particles traversing a given zone; E, particle energy; ΔE, change in energy of each zone due to radiative heat transfer; C, velocity of light; Δx and Δt, width of the zone and the sides of the time interval; Δ, thickness of the specimen; x and d, coordinate of the particle source and the distance the particle traverses before being absorbed; T_i and T_s, initial temperature of the specimen and the temperature of the bounding surfaces; $\tau_0 = \beta x$, optical thickness; $P = \beta K/(4n^2\sigma T_s^3)$, radiative–conductive parameter; $\Theta = T_s/T_i$, ratio of the temperature of the cold wall to the initial temperature of the specimen; n, index of refraction; c_2, spectral energy distribution constant in Planck's law; β_λ, spectral absorption coefficient.

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POROUS RADIATORS WITH SURFACE COMBUSTION IN THE FILTRATION OF A FUEL–OXIDANT MIXTURE

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A method is proposed for calculating porous radiators with the injection of fuel–oxidant mixture, taking the influence of all the basic parameters into account.

In [1], calculations were performed of porous radiators with the filtration through a permeable wall (porous or perforated) of liquid or gaseous fuel that reacts at the surface of the body with oxidant (oxygen) of the external medium. The case of filtration of a fuel–oxidant mixture is now considered, since porous radiators of this type are of broad practical application, but the method of calculation requires significant refinement.

The problem is formulated as follows.

Through a porous body of thickness $l = y_2 - y_1$ with internal energy sources or sinks q_V, a mixture of fuel and oxidant filters, with an initial temperature T_ε and a total filtration density j; $j = j_f + j_A$, where j is the transverse flux density of fuel and j_A is the transverse flux density of oxidant. This mixture burns at the "hot" surface of the wall, where $\bar{y} = \bar{y}_2 = 1$ in the oxygen filtering together with the fuel, the heat of combustion of which

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